

Dark energy interacting with two fluids

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A cosmological model of dark energy interacting with dark matter and another general component of the universe is investigated. We found general constraints on these models imposing an accelerated expansion. The same is also studied in the case for holographic dark energy.

I. INTRODUCTION

The existence of a dark component with an exotic equation of state [1] filling a flat universe [2] has been well established from the combined MAXIMA-1 [3], BOOMERANG [4],

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DASI [5] and COBE/DMR Cosmic Microwave Background (CMB) [6] observations. These results have been confirmed and improved by the recent WMAP data [7]. The accelerated expansion is consistent with the luminosity distance as a function of redshift of distant supernovae [8], the structure formation (LSS) [9] and the cosmic microwave background (CMB) [10]. This dark energy constitute the component with the biggest contribution to the energy density, responsible for 70 of the total energy density and has an equation of state similar to that of a cosmological constant, i. e., with a ratio $w = p/\rho$ negative and close to -1. Additionally, the other important contribution to the total density is the dark matter, which is roughly 1/3 of the total energy density, and is a gravitationally interacting form of non baryonic matter. Its existence have been well established by the observations of rotation curves in galaxies [11] and the CMB observations by WMAP [1], [10].

The cosmic observations show that densities of dark energy and dark matter are of the same order today. To solve this coincidence problem [12] (or why we are accelerating in the current epoch due that the vacuum and dust energy density are of the same order today ?) it is assumed an evolving dark energy field with a non-gravitational interaction with matter [13] (decay of dark energy to matter). In the case of the coupling between matter and quintessence it is motivated for string theory or arises after a conformal transformation of Brans-Dicke theory [14]. In general, it is assumed that the dark matter and dark energy are coupled by a term Q that gauges the transfer of energy from the dark energy to the dark matter. Interacting quintessence model, with a Q term proportional to the sum dark matter density and dark energy density, has been tested studying the effect of the coupling in the matter power spectrum, constraining the parameters of the models using the matter power spectrum measured by the 2-degree fiel galaxy redshift survey 2dFGRS [15]. The consequences of an interaction between dark matter and dark energy on the rates for the direct detection of dark matter were investigated at galactic level in [16]. In this case the interaction was modeled by assuming that the mass of the dark matter depends on the scalar field whose potential provides the dark energy. Very recently, the Abel cluster was investigated as an example of self gravitating system where interaction between dark energy and dark matter can be detected [17].

In the above discussion we have centered in the interaction between dark energy and dark matter. Nevertheless, it is physically reasonable and even expected from a theoretical point of view, that dark matter as well as dark energy can interact with other components of the

universe. See, for example, [18] for a model of dark energy interacting with neutrinos. If we focus in the possible decaying of dark energy, $w < -1$ is an indication that dark energy does indeed interact with another fluid. In [19] was studied the conditions under a dark energy can dilute faster or decay into the fermion fields. Kremer [20] has investigated a phenomenological theory of dark energy, with a given equation of state, interacting with neutrinos and dark matter.

Since a complete understanding of the nature of dark energy may involve to formulate a consistent theory of quantum gravity, not yet found, we make some attempts to clarify the nature of this energy following one of the new principles that emerges as a guideline to this ultimate theory, which is the holographic principle. This principle says that the number of degrees of freedom of a physical system should scale with its bounding area rather than with its volume. In the holographic dark energy models the investigations have been made in order to explain the size of the dark energy density on the basis of holographic ideas, derived from the suggestions that in quantum field theory a short distance cut-off is related to a long distance cut-off due to the limit set by the formation of a black hole [21]. In [22] the future event horizon was considered as the long distance in order to recover the equation of state for a dark energy dominated universe.

Following the previous results exposed above about the interactions between dark energy and dark matter and another component of the universe, our aim in the present work is to investigate the general constraints on a cosmological model where the interaction between dark energy, dark matter and another fluid, which we can not identified specifically, is considered. The will deduce restrictions for these models when the accelerated expansion is imposed and when, additionally, an holographic dark energy is taken account, which assume the following form

$$\rho_\chi = 3c^2 M_P^2 L^{-2} \quad (1)$$

where c is a numerical constant, $M_P = \frac{1}{\sqrt{8\pi G}}$ is the reduced Planck mass and L is a characteristic infrared cutoff. In this work we shall identified the Hubble horizon as L .

Our paper is organized as follows. In section I we present the model for a universe filled with dark matter, dark energy and another fluid. We shall impose that the interacting term Q and Q' which appears in the conservation equations are different. We derive general constraints for the difference between Q and Q' imposing an accelerated expansion. In

section II we discuss this cosmological scenario with an holographic dark energy and other constraints are presented. In section III we briefly discuss our results.

II. INTERACTING DARK ENERGY

In the following we modeled the universe made of CDM with a density ρ_m and a dark energy component, ρ_{DE} , which obey the holographic principle. We will assume that the dark matter component is interacting with the dark energy component, so their continuity equations take the form

$$\dot{\rho}_{DE} + 3H(\rho_{DE} + p_{DE}) = -Q', \quad (2)$$

$$\dot{\rho}_m + 3H(\rho_m + p_m) = Q. \quad (3)$$

Note that we have wrote Q and Q' in order to include the scenario in which the mutual interaction between the two principal components of the universe leads to some loss in other forms of cosmic constituents. In this case $Q \neq Q'$. Note we do not set a priori the sign of Q and $Q \neq Q'$, which means that we obtain the corresponding sign of this quantities from the constraints that we obtain bellow. If we denote this other component by ρ_X , its corresponding continuity equation is given by

$$\dot{\rho}_X + 3H(\rho_X + p_X) = Q' - Q. \quad (4)$$

We are taking about in this case that dark energy decay into dark matter (or viceversa, depending on the sign of Q) and other component. Of course, since our aim is to shed some light on the coincidence problem and the holographic principle, we expect to have $Q' - Q \ll 1$. The sourced Friedmann equation is then given by

$$3H^2 = \rho_{DE} + \rho_m + \rho_X - \frac{3k}{a^2}. \quad (5)$$

In the following we assume that $\rho_{DE} \gg \rho_X$ and $\rho_m \gg \rho_X$, which is consistent with the observable content of the universe, so ρ_X can be neglected from Eq. (5). Assuming that the equations of state for dark matter and dark energy are given by $p_i = \omega_i \rho_i$, where $\omega_i = \omega_i(H)$ ($i = 1, 2$), Eq. (2) and Eq. (3) can be written only in terms of the density

$$\dot{\rho}_{DE} + 3(1 + \omega_{DE})H\rho_{DE} = -Q', \quad (6)$$

$$\dot{\rho}_m + 3H[(1 + \omega_m)\rho_m + \pi] = Q'. \quad (7)$$

Adding Eq. (6) and Eq. (7) we obtain

$$\dot{\rho} + 3[(1 + \omega)\rho + \pi]H = 0, \quad (8)$$

where ρ , ω and π are defined by

$$\rho \equiv \rho_{DE} + \rho_m, \quad (9)$$

$$\omega\rho \equiv \omega_{DE}\rho_{DE} + \omega_m\rho_m, \quad (10)$$

and

$$\pi = -\frac{Q - Q'}{3H}. \quad (11)$$

Using Eqs. (8) and (5) and the definitions given by Eqs. (9), (10) and (11), we obtain

$$1 + \omega = -\frac{1}{H^2 + \frac{k}{a^2}} \left(\frac{2}{3}(\dot{H} - \frac{k}{a^2}) + \frac{\pi}{3} \right). \quad (12)$$

Let us first consider the constraint for π and Q derived from the condition of accelerated universe. The second Friedmann equation is

$$2\dot{H} + 3H^2 + \frac{k}{a^2} = -(p_{DE} + p_m + \pi). \quad (13)$$

Using Eq. (5) in Eq. (13) yields

$$\dot{H} = -\frac{1}{2}(1 + \omega_{DE} + (1 + \omega_m)r + \frac{\pi}{\rho_{DE}})\rho_{DE} + \frac{k}{a^2}. \quad (14)$$

The expression for the acceleration becomes

$$\frac{\ddot{a}}{a} = -\frac{1}{6}(1 + 3\omega_{DE} + (1 + 3\omega_m)r + 3\frac{\pi}{\rho_{DE}})\rho_{DE}, \quad (15)$$

where $r = \frac{\rho_m}{\rho_{DE}}$ is the ratio between dark matter and dark energy. For an accelerated universe, $\ddot{a} > 0$, the parameter π must satisfy the following inequality, which is independent of k ,

$$\pi < -\frac{1}{3}[(1 + 3\omega_{DE}) + (1 + 3\omega_m)r]\rho_{DE}. \quad (16)$$

If we evaluate π at the present time, taking $r \approx 0.42$ and $\omega_m = 0$, we obtain

$$\pi(t = 0) < -\frac{1}{3}[1.42 + 3\omega_{DE}]\rho_{DE}. \quad (17)$$

We have two possible scenarios: In the first one $1.42 + 3\omega_{DE} < 0$, which implies that π must be lower than a positive value. Then it is possible to take $\pi > 0$, or in other words $Q < Q'$, i.e., part of the dark energy density decays in dark matter and the rest in the other unknown energy density component, ρ_x . In this case $\omega_{DE} < -0.47$. In the second one $1.42 + 3\omega_{DE} > 0$, which implies that $\pi < 0$ or equivalently $Q > Q'$. In this case the dark matter received energy from the dark energy and from the unknown component and $\omega_{DE} > -0.47$. Obviously, observations favored the first scenario with decaying dark energy into dark matter.

If we assume that the unknown mechanism which leads to an interaction between dark energy and dark matter always implies $\pi > 0$ during the cosmic evolution, then in the non accelerated phase the Eq. (16) with $\omega_m = 0$ becomes

$$\pi(t) > -\frac{1}{3}[(1 + 3\omega_{DE}(t)) + r(t)]\rho_{DE}. \quad (18)$$

Since in our model the density of dark energy is a function of time we write $\omega_{DE}(t)$. Note that in our general approach we not assume a specific model for the dark energy and the interaction Q . In this case, the expression $1 + 3\omega_{DE}(t) + r(t)$ could be positive or negative. If it positive no new constraint is obtained for π . If it is negative we obtain a lower value for π .

Since we have not imposed a specific models for the dark energy and the interaction terms Q and Q' , we do not have any parameters that can be constrained by observations.

III. HOLOGRAPHIC DARK ENERGY

In the following we consider that the dark energy is given by

$$\rho_{DE} = 3c^2 H^2. \quad (19)$$

It is known that the without interaction between the two principal components in a flat universe ρ_{DE} behaves as ρ_m because of $\rho_m \sim H^2 \sim \rho_{DE}$ [22]. Let us discuss a general result which involves the curvature of the universe, in the framework of a holographic dark energy, filled with three interacting fluids, which implies $\pi \neq 0$, i.e., $Q \neq Q'$. The Eq. (5) can be rewritten as

$$1 = \frac{\rho_{DE}}{3H^2}(1 + r + s) + \Omega_k, \quad (20)$$

where $s = \rho_X/\rho_{DE}$ and $\Omega_k \equiv -k/a^2 H^2$. Using Eq. (19) in Eq. (20) we obtain

$$1 = c^2(1 + r + s) + \Omega_k. \quad (21)$$

For a flat universe the above equation tell us that $r + s$ must be constant, i.e., $\dot{r} + \dot{s} = 0$. If we assume the standard approach $Q = Q'$, an holographic dark energy implies that $r = \text{constant}$ for a flat universe. It is clear that depending, for example, in the choice that we can made to model Q , r can be variable during the cosmic evolution, obtaining a more suitable model to describe the cosmic coincidence.

In the following we discuss the contribution of π in order to obtain more negative ω_{DE} for the equation of state of the dark energy. For a flat universe with interaction Eq. (5) becomes (taking into account that $\rho_{DE} \gg \rho_X$ and $\rho_m \gg \rho_X$)

$$\rho_m = 3(1 - c^2)H^2. \quad (22)$$

Deriving the above equation and using Eq. (3) yields

$$\frac{2}{3} \frac{\dot{H}}{H^2} = \frac{Q}{9(1 - c^2)H^3} - (1 + \omega_m). \quad (23)$$

For a flat universe Eq. (12) becomes

$$\frac{2}{3} \frac{\dot{H}}{H^2} = \frac{\pi}{3H^2} - (1 + \omega). \quad (24)$$

Comparing Eq. (23) and Eq. (24) we obtain

$$\omega = \omega_m - \left[\frac{Q}{9(1 - c^2)H^3} + \frac{\pi}{3H^2} \right], \quad (25)$$

which means that if the matter content is described by dust, i.e., $\omega_m = 0$, and Hubble horizon as a characteristic length, the equivalent equation of the state for a universe with dark energy is different from zero. This result is obtained when exist $Q = Q' \neq 0$. In our case, a positive π , i.e., $Q < Q' \neq 0$, helps to a more negative value for ω . For a non flat universe the equation of state is given by

$$1 + \omega = \frac{1}{1 + k(aH)^{-2}} \left(1 + \omega_m - \frac{1}{3} \left[\frac{Q}{3(1 - c^2)H^2} + \frac{\pi}{H^2} - 2k(aH)^{-2} \right] \right) \quad (26)$$

Using the holographic dark energy we can obtain a new constraint for π , which involves the parameter c . For an accelerated universe, \ddot{a} , the constraint for π in terms of ω (defined by Eq. (10)), and ρ (defined by Eq. (9)) is given by

$$\pi < - \left(\omega + \frac{1}{3} \right) \rho, \quad (27)$$

which means that $\omega < -1/3$ for a positive π . Using the holographic condition given in Eq. (19) in Eq. (27) in $\omega < -1/3$, we obtain

$$\frac{\pi}{H^2} < 3c^2(|\omega| - \frac{1}{3})(1+r). \quad (28)$$

Evaluating the above inequality at the present time, where $|\omega| \approx 1$ and $r_0 \approx 3/7$ yields

$$\frac{\pi}{H^2}|_0 < \frac{20}{7}c^2. \quad (29)$$

Since $c^2 < 1$, in order to have a non zero dark matter density (see Eq.(22), the above equation indicates that $\pi \sim H_0^2$.

IV. DISCUSSION

In the present investigation we have considered a cosmological scenario where exist interactions between dark energy, dark matter and another component of the universe, which we do not identify explicitly. We have obtained general constraints, considering an accelerated expansion, on the parameter π . In the context of an holographic dark energy, and using the Hubble radius as infrared cutoff, we have shown that our scenario leads naturally, for a flat universe, to a more suitable approach to the cosmic coincidence problem in which r can be variable during the cosmic evolution. We also found, in this context, that a positive π , i.e., $Q < Q' \neq 0$, helps to obtain a more negative value for ω , as can be seen from Eq.(25). Finally, a constraint for π at the present cosmic time as been found, which can be of the order of H_0^2 . This constraint is obtained in the framework our the holographic approach and can be considered as only a first approximation to obtain a range for this parameter.

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